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Quantum Physics and Objective Description

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In the more refined description of concrete measuring procedures that is allowed by modern quantum theory, "objectivity elements" can be recovered that are usually deemed to be forbidden in quantum mechanics; actually, in the case of a *macrosystem*, a way towards an objective physical description appears.

1. INTRODUCTION

To easily understand the structure of modern quantum mechanics (Ludwig, 1983; Kraus, 1983; Davies, 1976; Holevo, 1982), one should first of all realize that the preparation of a quantum system (obviously in a statistical sense) is in general much better represented by a statistical operator on the Hilbert space **H** of the system than by an element $\psi \in \mathbf{H}$; so one has to shift the set **K** of the preparations from the space **H** to another suitable space **T** in which **K** gets a distinguished role: **T** is the Banach space of trace class operators on **H**; **K** is the convex subset of **T** consisting of the positive operators with trace 1; **K** generates **T** in the sense $A = A^\dagger \in \mathbf{T}$ can be represented as $A = \lambda_1 \varrho_1 - \lambda_2 \varrho_2$, $\lambda_{1,2} \in R^+$, $\varrho_{1,2} \in \mathbf{K}$, and taking the infimum of $\lambda_1 + \lambda_2$ on these representations, one gets the trace-norm $\|A\|_1 = \mathrm{Tr}[(A^+ A)^{1/2}] = \inf(\lambda_1 + \lambda_2)$; briefly, **K** is the base of the base-normed space **T**.

Any statistically determined transformation of a preparation is most naturally represented by an affine map $\mathscr{A}$ of **K** into **K**; then $\mathscr{A}$ can be uniquely extended as an endomorphism on **T**: $\mathscr{A}A = \lambda_1 \mathscr{A}\varrho_1 - \lambda_2 \mathscr{A}\varrho_2$, i.e., as a positive, isometric (and therefore trace-preserving) map on **T** into **T**. Let us assume that, connected to a transformation $\mathscr{A}^\gamma$ of the prepared system, a statistically determined event γ can be pointed out that can

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happen, γ^+, or not happen, γ^-; this is the most schematic description of a measurement. Then the decomposition $\mathscr{A}^\gamma\varrho=\sigma^\gamma_+ + \sigma^\gamma_-$ can be defined, where $\mathrm{Tr}(\sigma^\gamma_+)=p_\gamma$ is the probability that γ happens and $\varrho^\gamma_+/\mathrm{Tr}(\sigma^\gamma_+)$, $\varrho^\gamma_-/\mathrm{Tr}(\sigma^\gamma_-)$ are the two repreparations done taking also the facts γ^+ or γ^- into account. Now two affine maps $\mathscr{I}^\gamma_\pm$ are defined, $\mathscr{I}^\gamma_\pm\varrho=\sigma^\gamma_\pm$, which can be extended on **T** as linear, positive, contractive maps $\mathscr{I}^\gamma_\pm$ and that yield the decomposition $\mathscr{A}^\gamma=\mathscr{I}^\gamma_+ + \mathscr{I}^\gamma_-$. Let us borrow a basic concept from classical probability theory; in place of the two events $\gamma_\pm$, more generally we consider families of events linked to a measurable space (Ω, Σ) (Ω a set and Σ a σ-algebra of subsets of Ω), an event being labeled by an element of the σ-algebra Σ and any one of these elements taking the place of γ_+, γ_-. Then we have a whole family of maps $\mathscr{I}^\gamma(B)$, $B\in\Sigma$; this family is a positive-contraction-valued measure on Σ and gives a decomposition of the conservative transformation $\mathscr{A}^\gamma$ by

$$\mathscr{A}^\gamma=\mathscr{I}^\gamma(\Omega)=\sum_l \mathscr{I}^\gamma(\Omega_l) \tag{1.1}$$

for any partition of Ω into disjoint subsets Ω_l.

The physical interpretation is as follows; $\mathscr{A}^\gamma\varrho$ is the transformed preparation,

$$p^\gamma(B)=\mathrm{Tr}(\mathscr{I}^\gamma(B)\varrho) \tag{1.2}$$

is the probability of the event γ_B, and

$$\varrho^\gamma(B)=\frac{\mathscr{I}^\gamma(B)\varrho}{p^\gamma(B)}$$

is the repreparation done by taking γ_B happening into account. By the adjoint map $\mathscr{I}^{\gamma\prime}(B)$ on $\mathscr{B}(H)=\mathbf{T}'$, setting $F^\gamma(B)=\mathscr{A}^{\gamma\prime}(B)I$, equation (1.2) becomes

$$p^\gamma(B)=\mathrm{Tr}(F^\gamma(B)\varrho)$$

$F^\gamma(\cdot)$ is a positive operator valued measure on Σ, normalized by $F^\gamma(\Omega)=I$: it is an "observable" in the modern formulation; when $F^\gamma(\cdot)$ are projections and $\Sigma=\mathscr{B}(\mathscr{R}^s)$, the observable is associated to s commuting self-adjoint operators $X_l=\int_{\mathscr{R}^s} x_l\, dF^\gamma$, l: $1, 2, \ldots, s$; thus recovering the usual much more restrictive concept of observable. To sum up, all this comes out almost automatically; the shift $\psi\to\varrho$ starting the further shift: "operators transforming ψ" $\to$ "maps transforming ϱ."

A map $\mathscr{A}$ is reversible if $\mathscr{A}^{-1}$ exists, positive; then as it maps pure states (the extreme points of **K**) into pure states, it has the unitary structure $\mathscr{A}\varrho=X\varrho X^\dagger$, $X^\dagger X=I$, and no nontrivial decomposition (1.1) is possible; a reversible map cannot have a measuring character.

Let us dwell a little more on the transformations $\mathscr{A}$ of the system. Typically to $\mathscr{A}$ a time interval $[t_0, t_1]$ is associated; in the simplest situation the system is isolated, the transformation being its spontaneous dynamical evolution; the usual basic assumption is its reversibility: one has reversible maps $\mathscr{A}^{t}_{t_0}$ $(t_0 \le t \le t_1)$, with no measuring decomposition; this can be a precise formulation of the idea that the quantum mechanical description is nonobjective. If a system 1 is not isolated, its evolution in a fixed surrounding 2 in general depends on the whole history of the preparation, i.e., on $\varrho_1(t')$, $t' \le t_0$; in fact, these previous stages of system 1, recorded in the evolution of 2, provide by the interaction, memory effects for 1. However, there are many important cases in which these memory effects can be neglected and the system has a "Markovian" evolution:

$$\varrho_t = \mathscr{A}^{t}_{t_0}\varrho_{t_0} \tag{1.3}$$

where $\mathscr{A}^{t''}_{t_0}$ is a family of conservative irreversible maps with the structure

$$\mathscr{A}^{t'''}_{t'} = \mathscr{A}^{t'''}_{t''}\mathscr{A}^{t''}_{t'}, \qquad t_0 \le t' \le t'' \le t''' \le t_1$$

These maps allow measuring decompositions, as we shall describe in Section 2.

A prominent case in which memory effects should be negligible is when the system 1 interacts with a *measuring device*: then by suitable shielding, the interaction is just reduced to a well-protected probe-channel; e.g., 1 is an atom, 2 is the electromagnetic field + optical devices + photocounters. Another very general situation for a Markovian evolution is a system investigated over a time scale much longer than the decay time of the correlations in system 2, such as, e.g., a Brownian particle 1 in a liquid 2.

2. MARKOVIAN DYNAMICS AND MEASUREMENT

A family of Markovian conservative maps, assuming also complete positivity, is given for an infinitesimal time interval $[t, t + dt] \subset [t_0, t_1]$, by the general expression

$$\mathscr{A}^{t+dt}_{t}\varrho = [1 + \mathscr{L}(t)\,dt]\varrho$$

$$\mathscr{L}(t)\varrho = -\frac{i}{\hbar}[H(t), \varrho] + \sum_{j=1}^{s} \gamma_j^2(t)\{L_j(t)\varrho L_j^\dagger(t) - \tfrac{1}{2}[L_j^\dagger(t)L_j(t), \varrho]_+\} \tag{2.1}$$

where $H(t)$ is the Hamiltonian, $L_j(t)$ are operators on $\mathbf{H}$ describing irreversible processes, and $\gamma_j(t) > 0$ control their strength; one has $\mathrm{Tr}(\mathscr{A}^{t+dt}_{t}\varrho) = \mathrm{Tr}(\varrho)$ and one can see c-positivity and irreversibility writing the r.h.s. of (2.1) at first order in dt and observing that $dt > 0$ implies

$$\left[1 - dt\frac{i}{\hbar}H(t) - \sum_{j=1}^{s}\frac{1}{2}\gamma_j^2(t)L_j^\dagger(t)L_j(t)\right]\varrho$$
$$\times\left[1 + dt\frac{i}{\hbar}H(t) - \sum_{j=1}^{s}\frac{1}{2}\gamma_j^2(t)L_j^\dagger(t)L_j(t)\right]$$
$$+\sum_{j=1}^{s}\gamma_j^2(t)L_j(t)\varrho L_j^\dagger(t)\,dt > 0 \tag{2.2}$$

The s operators $L_j(t)$ which generate the irreversible dynamics also indicate the possible measuring decompositions of $\mathscr{A}_{t_0}^{t_1}$. These decompositions have been characterized in the context of "continuous measurement theory" initiated by Davies (1969, 1970, 1971) for the counting processes, developed in full generality by the Milan group (Barchielli *et al.*, 1982, 1983; Lupieri, 1983; Barchielli and Lupieri, 1985*a*,*b*; Barchielli, 1986*a*,*b*), and further investigated by Holevo (1988, 1989). The basic improvement with respect to the usual measurement theory consists of the fact that the time extension of the measurement can be taken into account; the outcome of the measurement not only consists of some fixed sets of values assumed by the observables (they may be practically constant) during the measurements, but it refers to the trajectories of the measured quantities. The space $\Omega_{t_0}^{t_1}$ is a space of trajectories; for any $[t', t''] \subset [t_0, t_1]$ a σ-algebra $\Sigma_{t_0}^{t_1}$ is given, related to the piece of trajectory corresponding to $[t', t'']$, the result of the measurement to be read as: the trajectory of the measured quantities belongs to a certain subset $B \in \Sigma_{t'}^{t''}$. The theory is based on a family of c-positive measures $\mathscr{I}_{t'}^{t''}$ on $\Sigma_{t'}^{t''}$ ["instruments" (Davies and Lewis, 1970)] which provide a decomposition of the dynamical map $\mathscr{A}_{t'}^{t''} = \mathscr{I}_{t'}^{t''}(\Omega_{t_0}^{t_1})$ and satisfy a composition law, (1.4):

$$\mathscr{I}_{t'}^{t'''}(B_2 \cap B_1) = \mathscr{I}_{t''}^{t'''}(B_2)\mathscr{I}_{t'}^{t''}(B_1), \qquad B_1 \in \Sigma_{t'}^{t''}, \quad B_2 \in \Sigma_{t''}^{t'''}$$
$$t_0 \leq t' \leq t'' \leq t'' \leq t_1 \tag{2.3}$$

Such a family $\mathscr{I}_{t'}^{t''}$ has been called an "*operation valued stochastic process*" in the work of the Milan group just cited; I am not going to give its explicit structure, but refer for a more extended account to the contribution of Prosperi in these proceedings (there $\mathscr{I}$ is replaced by $\mathscr{F}$).

Since sophisticated experiments can be done with some continuous monitoring of a quantum system, by this theory one can at least better understand the impressive success of experiments designed about the classical *history* of the measured quantum system; consider, e.g., the shelving effect for a trapped ion (Dehmelt, 1990); a beautiful description of such effect in this formalism was given in Barchielli (1987). The general relevance of the concept of a *consistent history* for a measured quantum system has been recently stressed by Omnés (1992).

There is, however, a subtle limitation in obtaining a naive classical description, as can be expected, due to the fact that this description is gained starting from measurement and not assumed *a priori*; actually only measurements on pieces (which may be very small) of trajectories make sense, the concept of a *classical state at a time point* being a further idealization. In fact the physically relevant quantities x_h related to a test function $h(t)$ cannot be represented as $x_h = \int_{t_0}^{t_1} dt\, h(t)x(t)$, $x(t)$ being the *underlying* state variable: $x(t)$ should be too irregular! Instead, choosing $h_\tau(t)$ as the characteristic function of the interval $[t_0, t_0+\tau]$, one considers the stochastic family $X_\tau = x_{h_\tau}$, which generates all the other quantities $x_h = \int_{t_0}^{t_1} h(\tau)\, dX_\tau$ by means of the increments of a nondifferentiable stochastic family X_τ. The σ-algebrae $\Sigma_{t'}^{t''}$ can be generated considering the increments

$$X_{\tau''} - X_{\tau'}, \qquad t' \leq \tau' \leq \tau'' \leq t''$$

of a set of classical stochastic processes

$$X_{j,\tau}^0, \quad X_{j,\tau}^1, \quad X_{j,\tau}^2, \quad j: 1, 2, \ldots, s$$

whose physical meaning and relationship with the operators $L_j(t)$ in equation (2.1) is elucidated by the expectation values

$$\begin{aligned}
\langle X_j^0(\tau'') - X_j^0(\tau')\rangle &= \int_{\tau'}^{\tau''} dt\, \mathrm{Tr}[L_j^\dagger(t)L_j(t)\varrho_t] \\
\langle X_j^1(\tau'') - X_j^1(\tau')\rangle &= \int_{\tau'}^{\tau''} dt\, \mathrm{Tr}[(L_j^\dagger(t) + L_j(t))\varrho_t], \qquad j = 1, 2, \ldots, s \qquad (2.4) \\
\langle X_j^2(\tau'') - X_j^2(\tau')\rangle &= \int_{\tau'}^{\tau''} dt\, \mathrm{Tr}\left[\frac{1}{i}(L_j^\dagger(t) - L_j(t))\varrho_t\right]
\end{aligned}$$

and by the second momenta (for simplicity only in the case of the same time interval)

$$\begin{aligned}
&\langle [X_{j_2}^{i_2}(\tau'') - X_{j_2}^{i_2}(\tau')][X_{j_1}^{i_1}(\tau'') - X_{j_1}^{i_1}(\tau')]\rangle \\
&= \int_{\tau'}^{\tau''} dt_1 \int_{\tau'}^{t_1} dt_2\, \mathrm{Tr}[[\mathscr{L}_{j_2}^{i_2}(t_1)\mathscr{A}_{t_2}^{t_1}\mathscr{L}_{j_1}^{i_1}(t_2) + \mathscr{L}_{j_1}^{i_1}(t_1)\mathscr{A}_{t_2}^{t_1}\mathscr{L}_{j_2}^{i_2}(t_2)]\varrho_{t_2}] \\
&\quad + \delta_{j_1 j_2}\delta_{i_1 i_2}\left[\delta_{i_1 0}\int_{\tau'}^{\tau''} dt \frac{1}{\chi_{j_1 0}^2(t)}\langle X_{j_1}^0(t)\rangle \right. \\
&\quad \left. + \delta_{i_1 1}\int_{\tau'}^{\tau''} dt \frac{1}{\chi_{j_1 1}^2(t)} + \delta_{i_2 2}\int_{\tau'}^{\tau''} dt \frac{1}{\chi_{j_1 2}^2(t)}\right] \qquad (2.5)
\end{aligned}$$

where the maps $\mathscr{L}^i_j(t)$ are

$$\mathscr{L}^0_j(t)(\cdot) = L^\dagger(t)(\cdot)L_j(t), \qquad \mathscr{L}^1_j(t)(\cdot) = L_j(t)(\cdot) + (\cdot)L^\dagger_j(T)$$

$$\mathscr{L}^2_j(t)(\cdot) = \frac{1}{i}(L_j(t)(\cdot) - (\cdot)L^\dagger_j(t)) \tag{2.6}$$

and the coefficients $\chi_{ji}(t)$ must satisfy the inequality

$$\chi^2_{j0}(t) + \chi^2_{j1}(t) + \chi^2_{j2}(t) \leq \gamma^2_j(t), \qquad j: 1, 2, \ldots, s \tag{2.7}$$

The other momenta have similar expressions and involve no more structure functions $\chi_{ji}(t)$: together with the operators L_j in (2.1), they fully characterize the measuring process. To take the expectation values implies a smoothing over the trajectories: in fact $\langle dX(t)\rangle$ is differentiable; one has

$$\frac{d\langle X^0_j(t)\rangle}{dt} = \mathrm{Tr}[L^\dagger_j(t)L_j(t)\varrho_t], \qquad \langle x^0_h\rangle = \int_{t_0}^{t_1} dt\, h(t)\, \mathrm{Tr}[L^\dagger_j(t)\varrho_t]$$

and the other analogous expressions; then it is clear that we are describing the trajectories of observables "associated" to the noncommuting operators:

$$L^\dagger_j(t)L_j(t), \quad L_j(t) + L^\dagger_j(t), \quad \frac{1}{i}(L^\dagger_j(t) - L_j(t)), \quad j: 1, 2, \ldots, s, \quad t\in[t_0, t_1]$$

all of them, at all $t\in[t_0, t_1]$, appearing as "compatible observables" encompassed by the observable: $F^{t_1}_{t_0}(B) = (\mathscr{I}')^{t_1}_{t_0}(B)I$. The maps $\mathscr{L}^i_j(t)$ in (2.6) depend on the operators $L_j(t)$ and yield typical structures arising in correlation functions; here these structures are a precise consequence of the formalism. Due to the $\chi_{ij}(t)$ terms in (2.5) the stochastic processes $X^i_j(\tau)$ are not differentiable; $X^{1,2}_j(\tau)$ look like Wiener processes and the irregular character of the *state variables* $x^{1,2}_j(t)$ is a white noise; $X^0_j(\tau)$ look like Poisson processes and the *state variables* $x^0_j(t)$ are affected by a shot noise. Different measurement procedures, i.e., different $\mathscr{I}^{t''}_{t'}$, can be associated to the same maps $\mathscr{A}^{t''}_{t'}$, as is indicated by the different choices of the coefficients $X_{ij}(t)$, implying different noises; e.g., if a radiating atom is observed through the radiated field, direct, homodyne, heterodyne detection is possible leading to different $\mathscr{I}^{t''}_{t'}$ (Barchielli, 1990, n.d.). Due to (2.7), the noises can be properly minimized, but not eliminated; they are related to the strengths of the irreversible terms in the dynamics. The classical level emerging for a system under measurement refers to Wiener and Poisson processes; in the more schematic theory of measurement, when the space of outcomes is the classical phase space, one can construct very useful instruments:

$$\mathscr{I}(B)\varrho = \int_B \frac{d^3x\, d^3p}{\hbar^3} |\alpha\rangle\langle\alpha|\varrho|\alpha\rangle\langle\alpha|$$

based on the classical measure $dx^3\,dp^3/\hbar^3$, in terms of the "instrument density": $|\alpha\rangle\langle\alpha(\cdot)|\alpha\rangle\langle\alpha|$, where $|\alpha(\mathbf{xp})\rangle$ is a coherent state. Similarly one can construct the instruments $\mathscr{I}_{t'}^{t''}(B)$ looking for suitable "instrument densities" on a space of classical Wiener and Poisson processes; I refer to the contribution of Barchielli in these proceedings for this interesting point of view.

3. MACROSCOPIC SYSTEMS

In Section 2 a quantum system undergoing a measurement in a time interval $[t_0, t_1]$ was shown to have a classical trajectory space as the direct consequence of its irreversible Markovian evolution; now we can read this result in another more general way: an open system with a Markovian dynamics (2.1) (by a suitable idealization, e.g.: too small time scales are excluded) has a classical trajectory space in which variables corresponding to the operators $L_j(t) + L_j^\dagger(t)$, $(1/i)(L_j(t) - L_j^\dagger(t))$, and $L_j^\dagger(t)L_j(t)$ are represented; their expectation values are given by the usual expressions

$$\mathrm{Tr}[(L_j(t) + L_j^\dagger(t))\mathscr{A}_{t_0}^{t}\varrho_{t_0}], \quad \text{etc.}$$

and their correlations by expressions (2.5) in which also unavoidable noise terms appear, determined by the functions $\chi_{ji}(t)$. These noises represent an effect of a specific environment, not included into the structure of $\mathscr{L}(t)$. Once also these functions $\chi_{ji}(t)$ are known the whole statistics of the stochastic processes $X_j^i(t)$, $t \in [t_0, t_1]$, is given and can be calculated in principle in terms of ϱ_{t_0}: e.g., the full statistics in $\Omega_{t_0}^{t_1}$ of an objective Brownian motion with quantum corrections was obtained (Barchielli, 1983), starting from the *phenomenological* $\mathscr{L}$ proposed by Lindblad (1976) for a Brownian particle.

Let us stress a peculiarity shown by equation (2.5): the limit of small coupling to the environment $\gamma_i(t) \to 0$ can be safely taken at the level of the expectation values (2.4), but makes all other momenta of the probability meaningless. Then one has the hint that general difficulties in the quantum theory of macrosystems have their roots in the usual assumption that the dynamics of an isolated microsystem is given by a reversible $\mathscr{A}_{t'}^{t''}$: such an assumption could be wrong, or admissible if only expectation values of some relevant observables are calculated. That quantum measurement theory can be well cured taking the environment into account has been stressed in particular by Joos and Zeh (1985) and Zurek (1981, 1982). Actually the very *definition of a system* implies a *separation procedure* (Lanz and Melsheimer, 1993) from the surrounding: in simple cases one expects that a separated system might be well described by a Markovian family $\mathscr{A}_{t'}^{t''}$, but only for a pure *microsystem* might $\mathscr{A}_{t'}^{t''}$ be reversible! To prepare an isolated system means that the quantum fields of its elementary components are confined inside definite space regions because of suitable boundary conditions, the environment being a *vacuum* for the outside fields. Due to the locality of field interaction, this description is not trivial and needs a reconsideration of current field theory. Also, an internal environment, representing the nondescribed structure of the *elementary* components, can play a role. One can expect that, due to the simplicity of the surrounding field vacuum, a Markovian $\mathscr{A}_{t'}^{t''}$ should give the dynamics of the isolated system: then one has the operators L_j in $\mathscr{L}$, the corresponding stochastic processes $X_j^i(t)$, and their trajectory space; these processes could take the place of the phase-space variables of classical statistical mechanics and have the role of *nonlocal hidden variables*, strongly dependent on the boundary conditions. A preliminary check of these ideas is done in Lanz and Melsheimer (1993): in a Galileian world, one-component systems are described taking $\mathbf{H}$ as the Fock space $\mathbf{H}_F$ of a quantum Schrödinger field $\psi(\mathbf{x})$; the *separation* of a system is done introducing one-particle energy eigenstates $u_n(\mathbf{x})$ $[u_{\mathbf{p}}(\mathbf{x})]$ for a particle confined inside (excluded from) a finite region Ω: $H^{(1)}u_n(\mathbf{x}) = W_n u_n(\mathbf{x})$, $\mathbf{x}\in\Omega$; correspondingly, annihilation operators

$$a_n = \int d^3x\, u_n^*(\mathbf{x})\psi(\mathbf{x}), \qquad a_{\mathbf{p}} = \int d^3x\, u_{\mathbf{p}}^*(\mathbf{x})\psi(\mathbf{x})$$

are introduced; $\varrho(t)$ is created by the operators $a_n^\dagger$ and is a vacuum for $a_{\mathbf{p}}$, i.e. $a_{\mathbf{p}}\varrho_t = \varrho_t a_{\mathbf{p}}^\dagger = 0$. For ϱ_t a Markovian dynamics is assumed, with

$$H_c = \sum_n W_n a_n^\dagger a_n + \frac{1}{2}\sum_{n_1n_2n_3n_4} a_{n_1}^\dagger a_{n_2}^\dagger V_{n_1n_2n_3n_4} a_{n_3} a_{n_4}, \qquad L_n = a_n$$

In this rough model the coefficients γ_n are not specified. In a more detailed treatment the separated system should be a *subdynamics* of the confined fields in a *vacuum* surrounding. By the choice

$$\chi_{n1} = \chi_{n2} \approx \frac{1}{\sqrt{2}}\gamma_n, \qquad \chi_{n0} \ll \chi_{n1}$$

the operation valued stochastic process $\mathscr{F}_{t_0}^t$ is given on the space of the huge set of stochastic processes $X_n^i(\tau)$ for all confined field modes n; for any quantity represented in usual statistical mechanics in terms of $a_n^\dagger$, a_n, $n = 1, 2, \ldots$, a corresponding stochastic variable can be constructed in a straightforward way in terms of the processes $X_n^i(\tau)$, $i = 0, 1, 2$, $n = 1, 2, \ldots$; the expectation values of bilinear expressions are the same as in usual quantum statistical mechanics (with dynamics generated by $\mathscr{L}$); in their correlation functions, and also in nonquadratic expressions γ_n-dependent terms arise; for any finite subset of variables the joint probability distribution can in principle be calculated. As an example, consider the operators for the confined field $\psi_c(\mathbf{x})$ and for the one-particle distribution $f(\mathbf{x}, \mathbf{p})$, given by

$$\psi_c(\mathbf{x}) = \sum_n a_n u_n(\mathbf{x}), \quad f(\mathbf{x}, \mathbf{p}) = \sum_{nn'} a_n^\dagger \langle n|F^{(1)}(\mathbf{x}, \mathbf{p})|n'\rangle a_n$$

where $F^{(1)}(\mathbf{x}, \mathbf{p})$, $\mathbf{x}, \mathbf{p} \in \mathbf{R}^6$, is a one-particle position-momentum observable (Lanz *et al.*, 1985); the corresponding stochastic variables at time t are given by $\tilde{\psi}_c(x) = \sum_n \tilde{a}_n(t) u_n(x)$ and by

$$\tilde{f}(\mathbf{xp}t) = \sum_{n \neq n'} \exp\left[\frac{i(W_n' - W_n)t}{\hbar}\right] \tilde{a}_{n'}^* \langle n'|F^{(1)}(\mathbf{xp})|n\rangle \tilde{a}_n(t)$$
$$+ \sum_n \frac{1}{\delta} [X_n^0(t+\delta) - X_n^0(t)] \langle n|F^{(1)}(\mathbf{xp})|n\rangle$$

where

$$\tilde{a}_n(t) = \frac{1}{\delta} \int_t^{t+\delta} \exp\left[\frac{iW_n\tau}{\hbar}\right] \frac{1}{2} d[X_n^1(\tau) - iX_n^2(\tau)]$$

Let me stress that $\tilde{f}$ is not a function of $\tilde{\psi}$; one does not have a *classical stochastic field theory*: a more complicated structure arises. The parameter δ determines the time scale; other parameters describing coarse graining in phase space are inside the structure of $F^{(1)}(\mathbf{xp})$; all these parameters strongly influence the correlation functions $\langle \tilde{\psi}(\mathbf{x}'t')\tilde{\psi}(\mathbf{x}t)\rangle$ and $\langle \tilde{f}(\mathbf{x}'\mathbf{p}'t')\tilde{f}(\mathbf{x}''\mathbf{p}''t'')\rangle$, the most critical term coming, respectively, from $\langle \tilde{a}_n^*(t)\tilde{a}_n(t)\rangle$ and from $\langle \tilde{a}_n^*(t)\tilde{a}_m^*(t)\tilde{a}_n(t')\tilde{a}_m(t')\rangle$, $n \neq m$. This model, which excludes electromagnetism, is not realistic enough to be taken too seriously: however, the results are qualitatively reasonable. The main job in statistical mechanics is to recognize the quantities that are not too wildly fluctuating: this formalism, due to the noise terms, displays such quantities in a very clear way and points to the hydrodynamic quantities. It can be seen the γ_n can be chosen small enough, e.g., $\approx 10^{-7}$, so that the Boltzmann equation for $\langle \tilde{f}(\mathbf{xp}t)\rangle$ still holds for reasonable time intervals, e.g., $\approx 10^5$ sec, without an appreciable noise for the hydrodynamic quantities related to space regions which are large enough, containing $\approx 10^{15}$ particles. At the kinetic stage the noise is not negligible.

Inside this description of isolated systems, *microsystems* appear in connection with the simplest breaking of the isolation condition. Take

$$\bar{\varrho}_{t_0} = \int d^3p\, d^3p'\, a_{\mathbf{p}}^\dagger \varrho_{t_0} a_{\mathbf{p}'} \langle u_{\mathbf{p}}|\varrho_{t_0}^{(1)}|u_{\mathbf{p}'}\rangle \tag{3.1}$$

where $\bar{\varrho}_{t_0}$ is a statistical operator in the Fock space $\mathbf{H}_F$ if $\varrho_{t_0}^{(1)}$ is a statistical operator on the one-particle Hilbert space $\mathbf{H}^{(1)}$ spanned by the improper eigenfunctions $u_\mathbf{p}$ of $\mathbf{H}^{(1)}$; call $\bar{\mathscr{I}}_{t'}^{t''}$ the previous $\mathscr{I}_{t'}^{t''}$ in which the Hamiltonian H_c is replaced by $H_c + \bar{V}$, $\bar{V}$ depending also on $a_\mathbf{p}$ and on $a_\mathbf{p}^\dagger$, thus making interaction with the environment possible; then one can write

$$\mathrm{Tr}_{\mathbf{H}_F}[\bar{\mathscr{I}}_{t_0}^{t_1}(B)\bar{\varrho}_{t_0}] = \mathrm{Tr}_{\mathbf{H}^{(1)}}[F^{(1)}[\bar{\mathscr{I}}_{t_0}^{t_1}(B), \varrho_{t_0}]\varrho_{t_0}^{(1)}] \tag{3.2}$$

where the *effect* operator $F^{(1)}$, i.e., $0 \le F^{(1)} \le I$, on $\mathbf{H}^{(1)}$ has been introduced; let me stress the striking reductions:

$$\mathbf{H}_F \to \mathbf{H}^{(1)}, \qquad \mathrm{Tr}_{\mathbf{H}_F}[\bar{\mathscr{I}}_{t_0}^{(1)}(B) \cdots] \to \mathrm{Tr}_{\mathbf{H}^{(1)}}[F^{(1)} \cdots]$$

and the reappearance of the formalism typical for a *microsystem*; here also the idealization $\varrho_{t_0}^{(1)} = |\psi_{t_0}\rangle\langle\psi_{t_0}|$ and the shift back $\mathbf{H} \to \mathbf{T}$ can be very effective. Now the perturbed system might be looked upon as a measuring device for the *microsystem*; obviously, as it is not so easy to construct a good detector, it is not easy to make $F^{(1)}[\bar{\mathscr{I}}^\gamma(B), \varrho_{t_0}] \neq p^\gamma(B)I^{(1)}$! Let us assume that $\bar{\varrho}_0$ given by (3.1) can be obtained by coupling to the first separated system a second one and taking the partial trace over its states; then also the source of the microsystem can be introduced, the microsystem becoming essentially a way to represent the most elementary interaction between these two, objectively described, separated systems: this is basically the concept of a microsystem developed by Ludwig (1985). However, the usual concept of microsystem is not recovered in a full way: the Schrödinger field

$$\psi(\mathbf{x}t) = \exp\left[\frac{iHt}{\hbar}\right]\psi(\mathbf{x}) \exp\left[\frac{-iHt}{\hbar}\right]$$

H being the *nonconfined* Hamiltonian, transforms covariantly under the unitary Galilei group; but due to the boundary conditions, covariance cannot be expected for $\mathscr{I}_{t_0}^{t''}(B)$, a_n, $a_\mathbf{p}$, $u_\mathbf{p}$; in this approach time translations and dynamical evolution are different concepts; only locally, and neglecting boundary effects, can covariance be recovered, e.g., $\mathbf{p}$ can be read as a momentum: a *thermodynamic limit* should be taken to define particles in a sharp way. A realization of this approach would attribute a peculiar central role to quantum electrodynamics, to be reconsidered taking *separation* of the systems into account. This description would be essentially incomplete since two environments are introduced: an external one, excluded by the boundary conditions, calling for large-scale physics; and an internal one, coming from the structure of nuclei. For these *self-separating* systems, objective properties could arise only by a more intrinsic dissipation mechanism.

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